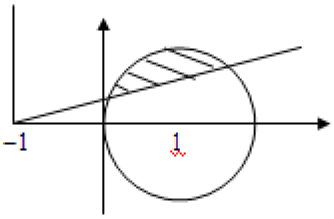
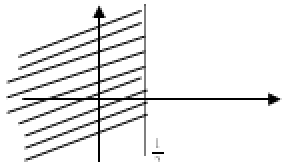
**MARK SCHEME**

**1.**

| | | |
|---|---|---|
| (i) (a)  | Circle | M1 A1 |
| | One half line correct | B1 |
| | Second half line | B1 (4) |
| | [s.c. Allow B1 for two "full" lines in correct position] | |
| (b) Shading correct region | | A1 ft (1) |
| (ii) (a) Rearrange $w = \frac{z-1}{z}$ to give $z = f(w)$ or $z - 1 = f(w)$ | | M1 |
| $\left(z = \frac{1}{1-w}, \Rightarrow\right)$ $z - 1 = \frac{w}{1-w}$, or $\lvert z-1 \rvert = \lvert z \rvert \lvert w \rvert \Rightarrow \lvert z \rvert \lvert w \rvert = 1$ | | A1 |
| Completion: ($\lvert z - 1 \rvert = 1 \rightarrow$) $\lvert w \rvert = \lvert 1 - w \rvert = \lvert w - 1 \rvert$ * | | A1 (3) |
| (b)  | Correct line shown | M1 |
| | Correct shading | A1 (2) |
| | | [10] |

**2.**

| | |
|---|---|
| (a) $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i \sin 5\theta$ | M1 |
| $(\cos\theta + i\sin\theta)^5 = \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$ | |
| $+ 10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$ | M1 A1 |
| $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ | M1 |
| $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$ | M1 |
| $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (*) | A1 cso (6) |
| (b) $\cos 5\theta = -1$ (or 1, or 0) | M1 |
| $5\theta = (2n \pm 1)180° \Rightarrow \theta = (2n \pm 1)36°$ | A1 |
| $x = \cos\theta = -1, -0.309, 0.809$ | M1 A1 (4) |
| | [10] |

**3.**

| | | |
|---|---|---|
| (a) | $\frac{r^2 - (r-1)^2}{r^2(r-1)^2} = \frac{2r-1}{r^2(r-1)^2}$ | M1, A1 (2) |
| (b) | $\sum_{r=2}^{n} \frac{2r-1}{r^2(r-1)^2} = \sum_{r=2}^{n} \frac{1}{(r-1)^2} - \frac{1}{r^2}$ | M1 |
| | $= \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \ldots + \frac{1}{(n-1)^2} - \frac{1}{n^2}$ | M1 |
| | $= 1 - \frac{1}{n^2}$ (*) | A1 cso (3) |
| | | (5 marks) |

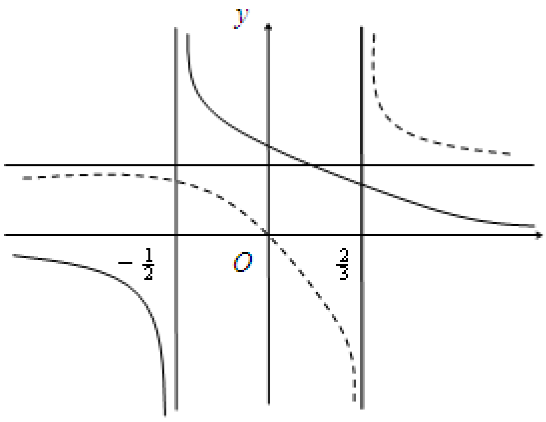
4.

| | | |
|---|---|---|
| (b) $\frac{dy}{dx} = x^2 - y^2 \Rightarrow \frac{d^2y}{dx^2} = 2x - 2y\frac{dy}{dx}$ | M1 A1 | |
| $\Rightarrow \frac{d^3y}{dx^3} = 2 - 2y\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^2$ allow at this stage | M1 A1 | (4) |
| (c) [ $y_{x=0} = 1$, $\left(\frac{dy}{dx}\right)_{x=0} = -1$, ] $\left(\frac{d^2y}{dx^2}\right)_{x=0} = 0 - 2(1)(-1) = 2$ | B1 | |
| $\left(\frac{d^3y}{dx^3}\right)_{x=0} = 2 - 2(-1)^2 - 2(1)(2) = -4$ | B1 | |
| Maclaurin: $y = 1 - x + x^2 - \frac{2}{3}x^3$ | M1 A1 | (4) |
| [ Alternative (c) $y = 1 + a_1x + a_2x^2 + a_3x^3$ | | [14] |
| $\Rightarrow x^2 - (1 + a_1x + a_2x^2 + a_3x^3)^2 = a_1 + 2a_2x + 3a_3x^2$ B1 | | |
| Compare coeffs $\Rightarrow a_1 = -1$; $a_2 = 1$, $a_3 = -\frac{2}{3}$. B1; M1 A1 ] | | |

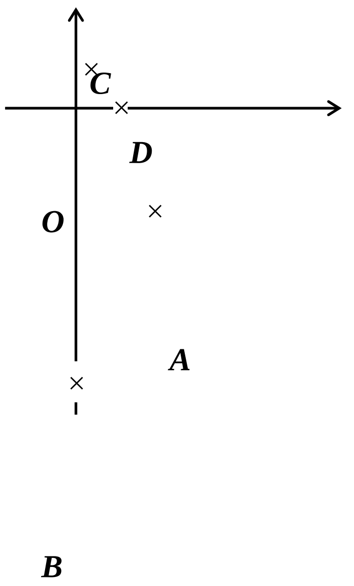
## 5.

| | |
|---|---|
| Identifying as critical values $-\frac{1}{2}$, $\frac{2}{3}$ | B1, B1 |
| Establishing there are no further critical values | |
| Obtaining $2x^2 - 2x + 2$ or equivalent | M1 |
| $\Delta = 4 - 16 < 0$ | A1 |
| Using exactly two critical values to obtain inequalities | M1 |
| $-\frac{1}{2} < x < \frac{2}{3}$ | A1 |
| | **(6 marks)** |
| Identifying $x = -\frac{1}{2}$ and $x = \frac{2}{3}$ as vertical asymptotes | B1, B1 |
| Two rectangular hyperbolae oriented correctly with respect to asymptotes in the correct half-planes. | M1 |
| Two correctly drawn curves with no intersections | A1 |
| As above | M1, A1 |

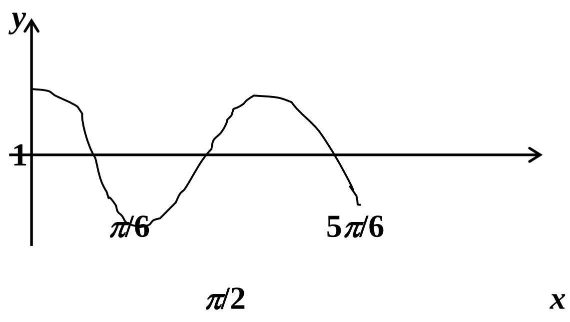
| | |
|---|---|
| Two correctly drawn curves with no intersections | A1 |
| As above | M1, A1 |



| Question Number | Scheme | Marks | |
|---|---|---|---|
| **12.** **(*a*)** | $v + x\frac{\mathrm{d}v}{\mathrm{d}x}$ **,= (4 + *v*)(1 + *v*)** | **M1, M1** | |
| | $x\frac{\mathrm{d}v}{\mathrm{d}x} = v^2 + 5v + 4 - v$ | **A1** | |
| | $x\frac{\mathrm{d}v}{\mathrm{d}x} = (v+2)^2$ ***** | **A1** | **(4)** |
| **(*b*)** | $\int \frac{1}{(v+2)^2}\,\mathrm{d}v = \int \frac{1}{x}\,\mathrm{d}x$ | **B1, M1** | |
| | $-\frac{1}{2+v} = \ln x + c$ **must have + *c*** | **M1 A1** | |
| | $2 + v = -\frac{1}{\ln x + c}$ | **M1** | |
| | $v = -\frac{1}{\ln x + c} - 2$ | **A1** | **(5)** |
| **(*c*)** | $y = -2x - \frac{x}{\ln x + c}$ | **B1** | **(1)** |
| | | | **(10 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| 13 | $z^2 = (3 - 3i)(3 - 3i) = -18i$ | M1 A1 (2) |
| (*b*) | $\frac{1}{z} = \frac{(3+3i)}{(3-3i)(3+3i)} = \frac{3+3i}{18} = \frac{1+i}{6}$ | M1 A1 (2) |
| (*c*) | $\lvert z \rvert = \sqrt{(9 + 9)} = \sqrt{18} = 3\sqrt{2}$ | |
| | $\lvert z \rvert = 18$ two correct | M1 |
| | $\left\lvert \frac{1}{z} \right\rvert = \sqrt{\frac{1}{18}} = \frac{1}{3\sqrt{2}} = \frac{\sqrt{2}}{6}$ all three correct | A1 (2) |
| (*d*) |  | |
| | two correct | B1 |
| | four correct | B1 (2) |
| (*e*) | $\frac{OB}{OD} = 18, \quad \frac{OA}{OC} = \frac{3\sqrt{2}}{\sqrt{2}/6} = 18$ | M1 A1 |
| | $\angle AOB = \angle COD = 45 \therefore$ similar | B1 (3) |
| | | (11 marks) |

| Question Number | Scheme | Marks |
| --- | --- | --- |
| `14. (a) | $y = \lambda x \cos 3x$ | |
| | $\frac{dy}{dx} = \lambda \cos 3x - 3\lambda x \sin 3x$ | M1 A1 |
| | $\frac{d^2y}{dx^2} = -3\lambda \sin 3x - 3\lambda \sin 3x - 9\lambda x \cos 3x$ | A1 |
| | $\therefore\ -6\lambda \sin 3x - 9\lambda x \cos 3x + 9\lambda x \cos 3x = -12 \sin 3x$ | |
| | $\lambda = 2$ cso | A1 (4) |
| (b) | $\lambda^2 - 9 = 0$ | M1 |
| | $\lambda = (\pm)3\text{i}$ | A1 |
| | $\therefore\ y = A \sin 3x + B \cos 3x$ form | M1 |
| | $\therefore\ y = A \sin 3x + B \cos 3x + 2x \cos 3x$ | A1 ft on $\lambda$'s (4) |
| (c) | $y = 1, x = 0 \Rightarrow B = 1$ | B1 |
| | $\frac{dy}{dx} = 3A \cos 3x - 3B \sin 3x + 2 \cos 3x - 6x \sin 3x$ | M1 A1ft on $\lambda$'s |
| | $2 = 3A + 2 \Rightarrow A = 0$ | |
| | $\therefore\ y = \cos 3x + 2x \cos 3x$ | A1 (4) |

| | | |
|---|---|---|
| (*d*) | <br><br>axes<br>shape | <br><br><br><br>B1<br>B1 (2)<br><br>**(14 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **15.** (*a*) | $\frac{1}{2}a^2\int 1+\cos^2\theta+2\cos\theta \quad d\theta$ | **M1 A1correct with limits** |
| | $= \frac{1}{2}a^2\int 1+\frac{\cos 2\theta+1}{2}+2\cos\theta \ d\theta$ | **M1 A1** |
| | $= 2\times\frac{1}{2}a^2\left[\theta+\frac{\sin 2\theta}{4}+\frac{\theta}{2}+2\sin\theta\right]_0^{\pi}$ | **A1** |
| | $= a^2\left[\frac{3\pi}{2}\right] = \frac{3\pi a^2}{2}$ | **A1** **(6)** |
| (*b*) | $x = a\cos\theta + a\cos^2\theta$<br>$r\cos\theta$ | **M1** |
| | $\frac{dx}{d\theta} = -a\sin\theta - 2a\cos\theta\sin\theta$ | **A1** |

| | | | |
|---|---|---|---|
| | $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0 \Rightarrow \cos\theta = -\frac{1}{2}$<br>**finding $\theta$** | M1 | |
| | $\theta = \frac{2\pi}{3}$ **or** $\theta = \frac{4\pi}{3}$ | | |
| | $r = \frac{a}{2}$ **or** $r = \frac{a}{2}$<br>**finding *r*** | M1 | |
| | $A$: $r = \frac{a}{2}$, $\theta = \frac{2\pi}{3}$ | | |
| | $B$: $r = \frac{a}{2}$, $\theta = \frac{-2\pi}{3}$<br>**both *A* and *B*** | A1 | (5) |
| (*c*) | $x = -\frac{1}{4}a \quad \therefore\ WX = 2a + \frac{1}{4}a = 2\frac{1}{4}a$ | M1 A1 | |
| (*d*) | $WXYZ = \frac{27\sqrt{3}a^2}{8}$ | B1 ft | (1) |
| (*e*) | **Area** $= \frac{27\sqrt{3}}{8} \times 100 - \frac{3\pi \times 100}{2} = 113.3\ \text{cm}^2$ | M1 A1 | (2) |
| | | | **(16 marks)** |